Evaluation of Open Channel Flow Equations

Introduction :

Most common hydraulic equations for open channels relate the section averaged mean velocity (V) to hydraulic radius (R) and hydraulic gradient (S). Some of these equations involve application of a roughness coefficient (e.g. Manning equation) or are based on a limited range of data (e.g. Lacey equation). Highwater observations at some sites can be used to calibrate roughness coefficients or provide confidence in the applicability of other equations. However, when limited site observations are available, judgment is required in knowing if certain equations are applicable or in the selection of a roughness coefficient. Published guidelines based on previous observations at certain sites are often used to assist in this judgment.

A large amount of hydraulic measurement data has been collected by the Water Survey of Canada at many sites in Alberta covering a wide range of channels and flow conditions. Coupling this data with stream slope estimates results in a large database that can be used to provide specific guidance on hydraulic calculations at these sites and to provide general guidance for use at other sites. In addition, this data can be used to evaluate the applicability of currently applied equations and possibly identify a new equation that may result in simplification and increased confidence in hydraulic calculations for natural channels.

Current Equations :

Most available hydraulic equations for open channels relate the section averaged mean velocity (V) to hydraulic radius (R) and hydraulic gradient (S). In wide open channels, R can be approximated by the mean flow depth (d), which is equal to the flow area (A) divided by the top width (T). In the absence of local hydraulic controls, the hydraulic gradient is usually equal to the channel slope for high in-bank flows.

Some equations also include a roughness parameter to account for the different nature of flow resistance offered by a range of channels. One of the most commonly applied equations is the Manning equation, which can be expressed as :

$$V = 1/n \, d^{2/3} \, S^{1/2}$$

where ‘n’ is a roughness coefficient. The power function approximations to logarithmic equations based on friction factor and roughness height (Henderson, 1966) yield a similar form for fully rough flow.

Guidance on selecting an appropriate roughness coefficient for a site is available in many publications, including Chow (1959), Fasken (1963), and Arcement and

Schneider (1989). Some of this guidance is in the form of example photographs and others based on channel classifications and descriptions. In addition, many studies such as Limerinos(1970), Parker and Peterson (1980), Bathurst et al (1981)), and Karim (1995) have been undertaken to try to relate the roughness parameter to physical parameters such as bed material size and bed configuration. Guidance on application of the Manning equation in hydraulic modelling of natural channels is covered in ASCE (1996), TAC (2001), HEC (2001), and AT(2001).

Bray (1979) refers to an equation published by Lacey which has similar form to the Manning equation, but without a roughness parameter. This equation, which was based on data collected in India on sand bed canals, can be expressed as :

$$V = 10.8\ d^{2/3}\ S^{1/3}$$

It is noted in Blench (1966) that Lacey preferred simple exponents rather than multiple decimal least squares fit exponents as long as they provided reasonable fits to available data. This is apparently based on the belief that most physical processes are governed by relatively simple laws. Lacey and Pemberton (1972) state that the Lacey equation is applicable to coarse sand bed channels. They also suggest that this equation is part of a family of equations covering the entire spectrum of sediment sizes.

Bray (1979) used data collected for many gravel bed streams in Alberta to evaluate the applicability of these equations. It was concluded that the Lacey equation performed as well as the roughness parameter equations for high in-bank flows in alluvial channels.

Hydraulic Measurement Data

The Water Survey of Canada (WSC) has collected and published flow data at many sites in Alberta since the early 1900's. There are over 1000 sites in their database, although the number of active gauges at any one time has never exceeded 500. The streams in the database range in width from a few metres to more than 600m. Due to the presence of the Rocky Mountains and prairies in Alberta, a wide range of channel slopes have also been covered by these stream gauges. The range of streambeds includes gravel, sand, and silt.

Many significant floods have occurred in Alberta since these gauges have been in operation. The largest floods in the foothills areas tend to be caused by rain storms, while most of the largest floods in the prairie areas have been related to snowmelt events. The existence of stream gauging measurements under relatively high flow conditions depends on factors such as: the location of active gauges with respect to the storms, the ability of WSC staff to mobilize during the sometimes brief runoff response period, and the ability to safely and accurately take gauging measurements under sometimes dangerous flood conditions.

Although the published WSC database shows mainly daily flow values, these data are based upon rating curves developed from actual stream gauging measurements. Actual stream gauging records for active stations are retained in the regional WSC offices and site visit summary records are also maintained. These summaries include the measurement date, 'T', 'A', stage (Y), and flow (Q). From this data, 'V' and 'd' can be easily calculated for all measurements.

In this study, site visit summary data has been collected for measurements taken during the years of highest flows for all active gauges in Alberta on natural channels, resulting in 308 sites.

An additional benefit to compilation of the gauged hydraulic data is the ability to create rating curves (Y vs. Q) for each site in the study. This data has been added to the *PeakFlow* tool developed by Alberta Transportation to assist in querying the peak flow database. This tool can now plot a rating curve for each station in the compiled database, showing the actual points at which the river was gauged. The key hydraulic data for each of these gaugings (T, A, Y, Q) is also displayed. In addition to confirming hydraulic calculations for these sites, this information can be used to identify the amount of extrapolation that may have occurred in the published peak flow data. Also, the peak stage corresponding to the published peak flow values can be estimated.

Stream Slope Estimation

The hydraulic slopes for these gauged sites have not been published by WSC. Slope values have been measured from surveys at some of the sites as published in Kellerhals et al (1972) and from bridge site surveys for those gauges located close to bridges. However, many of these surveys are of insufficient length to show the full context of the stream profile or to provide accurate assessment of the governing hydraulic slope at the site.

Stream profiles have been added to the Hydrotechnical Information System (HIS) tool developed by Alberta Transportation. These profiles were extracted from DTM data (SRTM – 3 arc second spacing) using the Global Mapper GIS tool and the provincial stream arcs GIS layer. The HIS tool locates bridges and WSC gauges on these profiles, facilitating the estimation of the channel slope at these locations.

These DTM based channel profiles provide enough data to pick up the effective hydraulic slope at most sites, with the exception of those located near a sharp change in profile. Stream slopes derived in this manner agree quite well with those reported at coincident sites by Kellerhals et al (1972), where surveys were generally quite extensive. Less agreement was found with comparison to the slopes derived from bridge site surveys, especially for low gradient channels, due to the limited length of profile surveyed.

Database Compilation :

Plots of 'V' vs. 'd' were prepared for each gauge site in the database. In most cases, a relationship between 'V' and 'd' was observed, following a power function form :

$$V = Cd^{m}$$

where 'm' generally was in the range of 0.5 to 0.67. A best fit 'V' in the vicinity of the largest recorded 'd' was identified by visually fitting this form of equation to the data. Of the 308 sites analysed, about 250 showed a significant relationship between 'V' and 'd'. The lack of an observed relationship for the remaining sites is likely due to the lack of measurements under high flow conditions. An example of the 'V' vs. 'd' plot for one of the sites is shown in Figure 1.

The scatter in the data on these plots is likely due to the many natural open channel flow phenomena that affect the stage discharge relationship, such as moving bedforms and dynamic channel geometry, beaver dams, drift jams, ice cover influence etc. More weight was given to higher flow data points, as these are more likely to be under uniform flow conditions and free from low flow effects such as pool and riffle sequence effects.

Slope estimates were developed for all gauge sites as discussed above. Reasonable slope estimates were derived for about 250 of the 308 sites. Those with poor slope estimates were generally due to a sudden change in profile in the vicinity of the gauge or the presence of downstream hydraulic controls such as a confluence.

The values for slope and coefficient 'C' were then assembled into a database for analysis.

Analysis :

A power function equation of similar form to the Manning and Lacey equations was fit to the entire dataset. The resulting best fit equation was :

$$V = 13.3d^{0.69}S^{0.39}$$

with a correlation coefficient of 0.93. This fit suggests shows a strong correlation and indicates that 'd' and 'S' are the most important parameters in predicting 'V'. The exponent for 'd' is very close to the $^{2}/_{3}$ value of both the Manning and Lacey equations, and the 'S' exponent lies between the published exponents for these two equations.

For direct comparison with the Lacey equations, a best fit equation was developed while holding the 'd' exponent to 0.67 and the 'S' exponent to 0.33. The resulting equation coefficient is 9.1, which compares with the published value of 10.8. The correlation coefficient dropped to 0.92. A similar process for the Manning equation resulted in a best fit over the dataset with 'n' = 0.039 and a correlation coefficient of 0.89.

To visualize the fit to data, the coefficient 'C' was calculated for exponent 'm' = 0.67 and then plotted vs. $S^{0.33}$ (Lacey, see Fig. 2) and $S^{0.5}$ (Manning, see Fig. 3). Both plots show a significant correlation between 'C' and 'S'. However, each plot shows that a best fit to the data intercepts the axes at opposite sides of the origin. This suggests that an intermediate value of the slope exponent may provide a better fit to the data over the entire range of slopes. This is confirmed by plotting 'C' vs. $S^{0.4}$, as suggested by the best fit equation (see Fig. 4). On this plot, a best fit line intercepts the origin and provides a better fit over the range of slopes observed. The resulting best fit equation, with slightly simplified exponents is :

$$V = 14d^{0.67} S^{0.4}$$

An additional measure of the fit to the dataset is to count the number of data points that fall within a certain percentage of the best fit. A plot over the observed range can then be used to compare the quality of the various fits and the sensitivity to exponents. Fig. 5 shows the percentage of data points falling within a certain range of the best fit for a range of 'd' exponents (0.6, 0.67, and 0.75) with the slope exponent fixed at 0.4. This plot shows that the results are not very sensitive to the 'd' exponent, with a value of 0.67 appearing to be the most suitable. Fig. 6 shows a similar plot, but with a range of 'S' exponents (0.33, 0.4, and 0.5) with the 'd' exponent held fixed at 0.67. This plot provides a comparison of the Lacey, Manning, and $S^{0.4}$ best fit equations as all have a 'd' exponent of 0.67. Although all three equations provide reasonable fits to the data, the equation with $S^{0.4}$ again appears to provide the best fit.

These plots indicate that about 60% of the data points fall within 10% of the $S^{0.4}$ prediction equation and that 90% of the data points fall within 20% of the $S^{0.4}$ prediction equation. Given the scatter of the 'V' vs. 'd' plots and the inaccuracy of the slope estimates, this can be considered a tight fit. Some additional scatter is expected due to the variability in cross sections within a natural channel reach and the degree to which the gauging section is representative of the overall reach. It is likely that the data points that appear as outliers on the plots are either under the influence of hydraulic controls other than the channel slope or have not recorded sufficient high in-bank flows to accurately display the 'V' vs. 'd' relationship for that site.

Data on this plot were investigated further to see if other trends or groupings based on other parameters such as channel width, sinuosity, or maximum mean depth observed could be found. However, no such trend was discovered. It is likely that the relationship between V, d, and S is so dominant that the accuracy of the data is insufficient to identify secondary trends.

Many publications have shown lower values of n (0.025 to 0.03) for large, relatively flat rivers, and larger values (0.04 to 0.05) for smaller steep rivers. Equating the new $S^{0.4}$ equation with the Manning equation yields:

$$n = S^{0.1}/14$$

This equation results in n = 0.028 for S = 0.0001, n = 0.036 for S = 0.001, and n = 0.045 for S = 0.01. This is consistent with the published recommendations and suggests that the need for a roughness coefficient may actually be due to the required correction for the slope exponent.

Conclusion :

The dataset compiled from WSC measurements and DTM based hydraulic slope estimates shows a strong correlation between 'V' and the parameters 'd' and 'S'. Best fits to the data suggest that the 'd' exponent used in the Lacey and Manning equations is suitable. However, an 'S' coefficient of 0.4 appears to provide a better fit over the range of observed slopes than the exponents of the Lacey and Manning equations. The data suggests that much of the variance in the Manning roughness coefficient observed in natural channels can be attributed to the slope exponent.

Based on the available data, the following equation is recommended for application in natural channels where normal flow (hydraulic gradient equals channel slope) appears to be the hydraulic control:

$$V = 14d^{0.67}\,S^{0.4}$$

This equation predicts 'V' based on 'd' and 'S' within 10 to 20% for the majority of sites and should be sufficient for most one dimensional, steady flow calculations for natural channels in engineering applications, without the addition of a roughness coefficient.

There is very little data in this study for small vegetated bed channels. Therefore, it is suggested that traditional methods, such as the Manning equation with published roughness coefficient guidance, still be applied to these channels. It should also be noted that the 'V' vs. 'd' relationships observed in the database are based on relatively high in-bank flows and that significant scatter was observed at lower flows due to the range of hydraulic influences that may govern in this range.

References :

ASCE, “River Hydraulics”, Adapted From US Army Corps Of Engineers No. 18, 1996.

AT, “Practical Hydraulic Modelling Considerations”, Alberta Transportation, 2001.

Arcement, G. J., Schneider, V. R., “Guide For Selecting Manning’s Roughness Coefficients For Natural Channels And Floodplains”, US Geological Survey, Water Supply Paper 2339, 1989.

Bathurst, J.C., Li,, R., Simons, D.B., “Resistance Equation For Large Scale Roughness”, Journal Of The Hydraulics Division, ASCE, Vol. 107, No. HY12, Proc. Paper 16743, Dec. 1981, pp. 1593-1613.

Blench, T., *Mobile-Bed Fluviology*, The University Of Alberta Press, 1969.

Bray, D.I., “Estimating Average Velocity In Gravel-Bed Rivers”, Journal Of The Hydraulics Division, ASCE, Vol. 105, No. HY9, Proc. Paper 14810, Sept. 1979, pp. 1103-1122.

Chow, V.T., *Open Channel Hydraulics*, McGraw-Hill Book Co., New York, N.Y., 1959.

Fasken, G. B., “Guide For Selecting Roughness Coefficient ‘n’ values for Channels”, Soil Conservation Service, USDA, 1963.

Henderson, F.M., *Open Channel Flow*, MacMillan Co., New York, N.Y., 1966.

Karim, F., “Bed Configuration And Hydraulic Resistance In Alluvial-Chanel Flows”, Journal Of The Hydraulic Engineering, ASCE, Vol. 121, No. 1, Paper No. 5739, Jan. 1995, pp. 15-25.

Kellerhals, R., Neill, C.R., and Bray, D.I., “Hydraulic and Geomorphic Characteristics Of Rivers In Alberta”, River Engineering and Surface Hydrology Report 72-1, Research Council Of Alberta, Edmonton, Canada, 1972.

Lacey, G., and Pemberton, W., “A General Formula For Uniform Flow In Self-formed Alluvial Channels”, *Proceedings of the Institution of Civil Engineers*, Technical Note 71, 1972.

Limerinos, J. T., “Determination Of The Manning Coefficient From Measured Bed Roughness In Natural Channels”, Water Supply Paper 1898-B, U.S. Geological Survey, Washington, D.C., 1970.

Parker, G., and Peterson, A. W., “Bar Resistance Of Gravel-Bed Streams”, Journal Of The Hydraulics Division, ASCE, Vol. 106, No. HY10, Proc. Paper 15733, Oct. 1980, pp. 1559-1575.

TAC, “Guide To Bridge Hydraulics”, Transportation Association Of Canada, 2001.

USACE, “HEC-RAS Hydraulic Reference Manual”, Hydrologic Engineering Center, US Army Corps Of Engineers, 2001.

Water Survey of Canada, Calgary District Office, “Velocity And Cross Section Data For Selected Alberta Stream Gauging Stations”, July 1977.

Appendix :

Figure 1 – V vs. d Plot For North Saskatchewan River At Edmonton

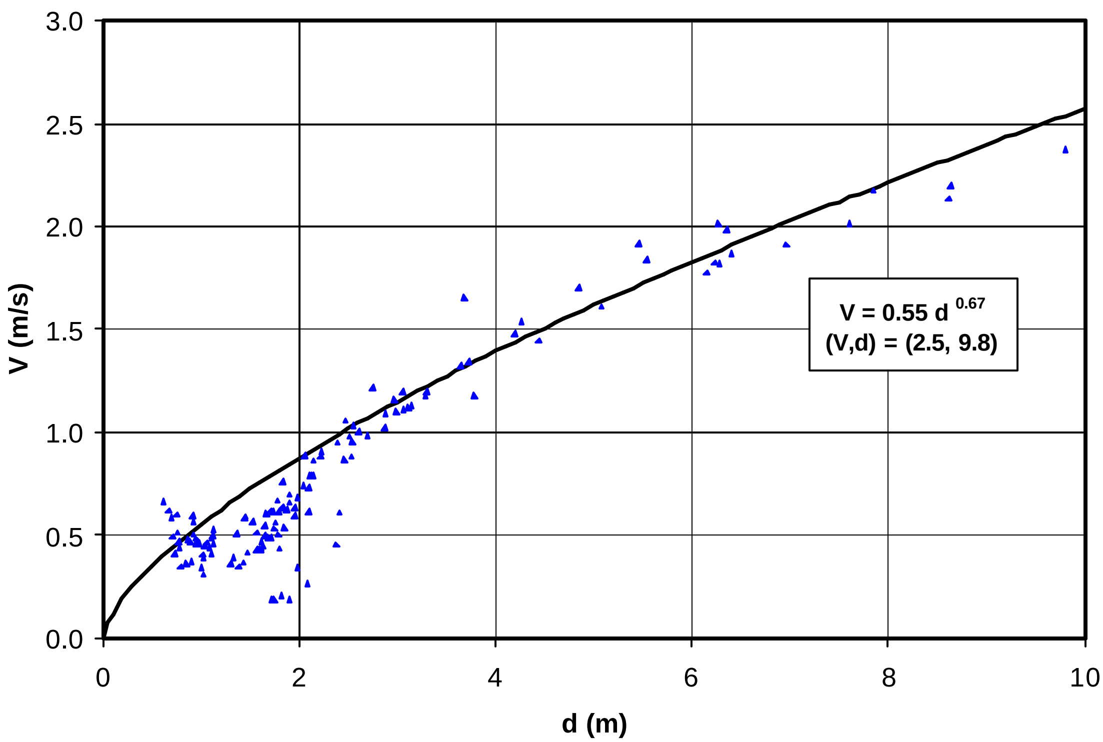

Figure 2 - Comparison With Lacey Equation

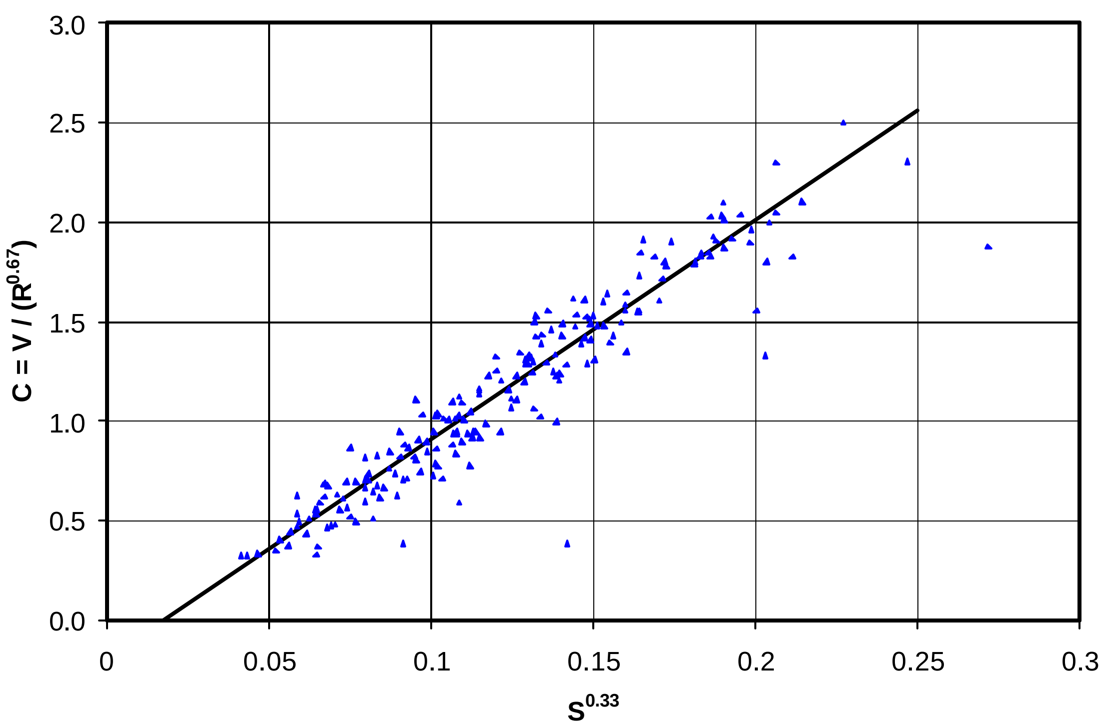

Figure 3 - Comparison With Manning Equation

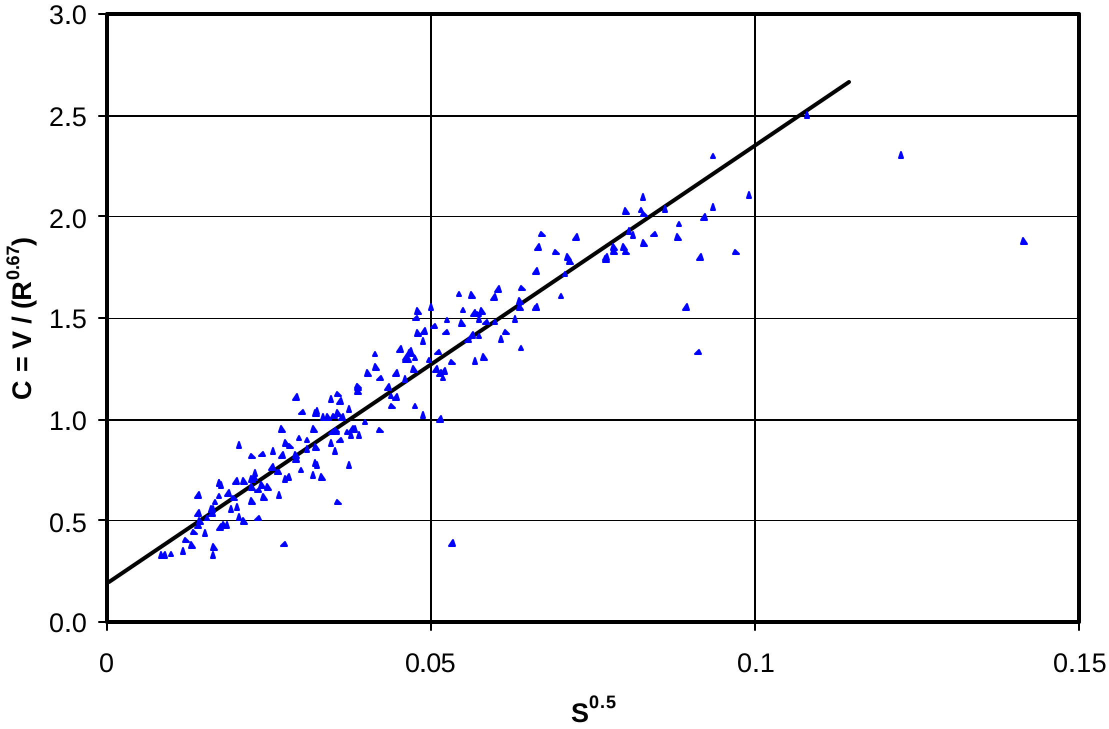

Figure 4 – Slope Exponent = 0.4

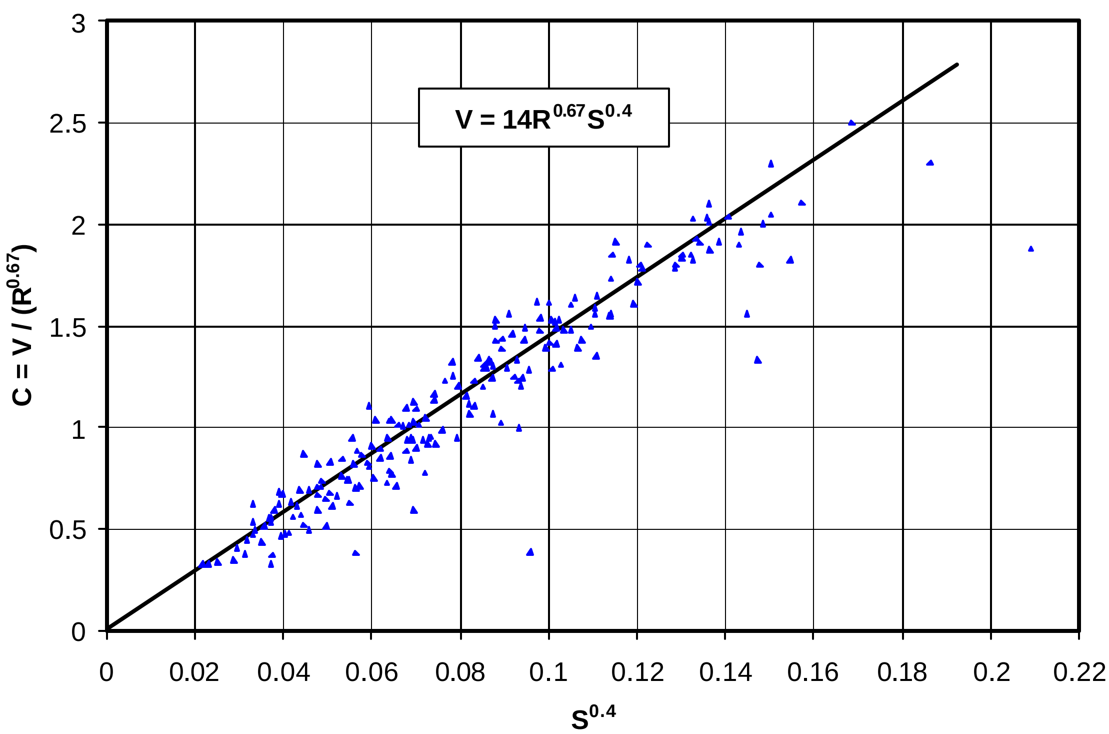

Figure 5 – Range of Fit – $S^{0.4}$

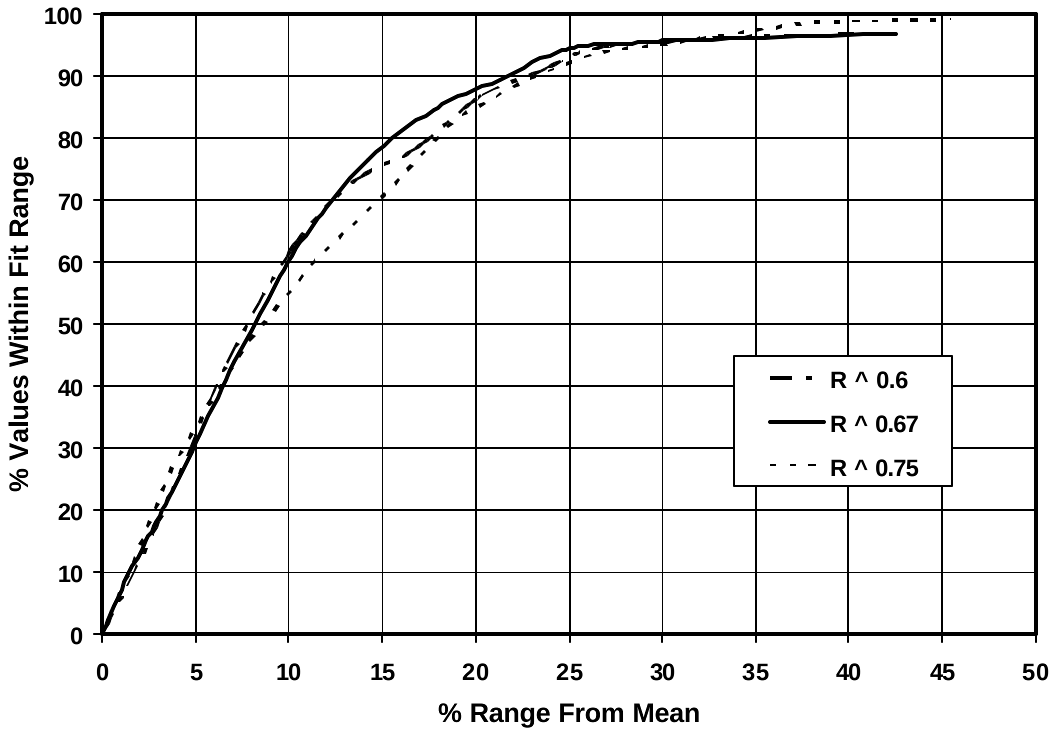

Figure 6 – Range of Fit – $R^{0.67}$